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THE ANALYTICAL TREATMENT OF SECONDARY FLOWS AND ASSOCIATED LOSSES IN AXIAL-FLOW TURBOMACHINES

by

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Abstract:

This report presents the current state of the art of the analytical treatment of secondary flows and associated losses in axial-flow turbomachines. A physical description of the secondary-flow phenomena is given, as well as the theoretical methods that are available to treat secondary flows and to predict secondary losses.

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I. INTRODUCTION

The existence of secondary flows in certain fluid-flow configurations has been recognized for some time. In 1927 W. R. Dean [Ref. 1] developed a theoretical analysis of secondary flow for laminar fluid flow in curved pipes with large ratios of bend radius to pipe diameter. Due to the complexity of the flow in turbomachines, however, such an analysis is too restrictive to be directly applicable.

The modern design methods for axial-flow turbomachines that use a quasi-three-dimensional through-flow calculation require, nevertheless, the radial distribution of the loss in total pressure, which is usually obtained by consideration of two-dimensional cascade data, either experimental or theoretical, and three-dimensional flow effects, the secondary flow. In the present era of small, high-specific-energy turbomachines where secondary flows can account for as much as 80% of the loss in total pressure [Ref. 2], the prediction of the secondary losses is especially critical.

The object of this report is to present the current state of the art of the analytical treatment of secondary flows and associated losses in axial-flow turbomachines. To that end a physical description of the secondary-flow phenomena is given, as well as the theoretical methods that are available to treat secondary flows and to predict secondary losses.

II. SECONDARY FLOW DEFINED

Secondary flow in a turbomachine is defined as the difference between the actual flow and the idealized axisymmetric flow, and it is principally caused by the interaction of the hub and casing boundary layers with the main through flow. Similarly, secondary flow in a rectilinear cascade is considered to be the difference between the real flow and the two-dimensional flow, and it is due to the interaction of the end-wall boundary layers with the main through flow.

In order to obtain a better understanding of the complicated secondary-flow phenomena, several investigators have used various flow-visualization schemes to observe the flow in plane and annular cascades of compressor and turbine bladings. Extensive experiments were conducted by NACA (now NASA) utilizing smoke, hydrogen-sulfide traces on white lead paint, and nondrying paint [Refs. 3-9]. A lampblack method was used by Fritzsche [Ref. 10], a particle-track method by Armstrong [Ref. 11], the hydrogen-sulfide technique by Louis [Ref. 12], and titanium-dioxide traces by Griepentrog [Ref. 2].

Flow visualization, of course, only gives qualitative results; but a fairly clear picture of secondary flow has resulted, as shown in Fig. 1. The secondary flows and vortices of Fig. 1 are closely interrelated and are part of the total flow field; but in order to successfully treat their effects experimentally and theoretically, they are separated for convenience into the following: cascade secondary flow, corner vortices (not shown in Fig. 1), trailing vortices, tip-clearance flow, scraping effect, and radial flow.

A. CASCADE SECONDARY FLOW

The explanation of the formation of the cascade secondary vortices is based on Vavra [Ref. 14] with the flow as depicted in Fig. 2. The passage between two neighboring blades of a cascade without diffusion or expansion is represented by a constant-section bend. A flow with uniform static pressure enters the bend from the left as indicated in the figure. A nonuniform total-pressure distribution results from the boundary layers that are formed at the upper and lower walls ahead of the bend. Hence, the inlet velocity profile has a uniform part where $V_1 = V_{1m}$, a constant, and goes to zero at both end walls.

The flow through the bend is assumed to be steady, incompressible, and inviscid; therefore, the equation of motion is

$$\nabla(P_t/\rho) = \vec{V} \times (\nabla \times \vec{V}) \tag{1}$$

where P_t is the total pressure, ρ is the density, and $\vec{V}$ is the velocity vector. In the absence of friction, $V_1 = V_{1m}$ is taken as constant along the entire flow path. With the coordinate system of Fig. 2, $\vec{V} = \vec{i}_1 V_{1m}$ and $V_2 = V_3 = 0$; thus, neglecting the changes in the z-direction and the gravitational forces, Eq. 1 becomes

$$\nabla\left(\frac{p}{\rho} + \frac{V_{1m}^2}{2}\right) = \vec{i}_1 V_{1m} \times \frac{\vec{i}_3}{R}\left[-\frac{\partial(RV_{1m})}{\partial R}\right] = \vec{i}_2 \frac{V_{1m}}{R}\frac{\partial(RV_{1m})}{\partial R} \tag{2}$$

The velocity V_{1m} is also constant in the radial direction outside the wall boundary layers; so at the point P where $R = R_{cm}$, there is

$$-\nabla(p/\rho) = -\vec{i}_2 V_{1m}^2/R_{cm} \tag{3}$$

where p is the static pressure. The quantity $-\nabla(p/\rho)$ is the force per unit mass exerted on a fluid particle.

Consequently, the fluid particles are in equilibrium between the centrifugal force and the static pressure gradient along a radius between the curved channel walls, where the convex wall and the concave wall are analogous to the cascade-blade suction side and pressure side respectively. Since the static pressure inside a boundary layer equals that outside of it, the same gradient $\nabla(p/\rho)$ is assumed to exist in the direction of $\vec{i}_3$ through P from the lower to the upper wall. At the point P_u the velocity V_{1u} is smaller than V_{1m}; and since the force per unit mass $-\nabla(p/\rho)$ exerted on a fluid particle at P_u is equal to that at P, the radius of curvature R_{cu} of the flow path at P_u must be smaller than R_{cm} to satisfy the equilibrium condition of Eq. (3). Similar conditions exist at the point P_l. Therefore, the fluid particles at the two end walls have the tendency to flow from the pressure side of the channel to the suction side, and the movement continues on the suction side towards mid-channel. Since the flow directions are opposite at the upper and lower walls, two cascade secondary vortices S_u and S_l are formed with opposing rotations as shown in Fig. 2.

B. CORNER VORTICES

If viscous effects are considered in the flow of Fig. 2, corner vortices will be formed at the intersections of the end walls and the suction surface of the blading due to the interaction of the accumulation of low-energy boundary-layer material flowing into this corner with the blade suction-side boundary layer. The corner vortices are shown in Fig. 3, where the cascade secondary vortices are omitted for clarity.

Senoo [Ref. 16] hypothesized the formation of the corner vortices by representing the end-wall boundary layer by a series of vortex sheets. The deflection of a vortex sheet in the corner between the end wall and the blade suction side causes a self-induced rolling-up motion, and the secondary flow in the corner is not a single vortex but a rolled-up vortex sheet. Senoo obtained good qualitative agreement between this hypothesis and measurements of the boundary layer on an end wall of a turbine stator cascade.

C. TRAILING VORTICES

The vortices that exist at the exit of the cascade as trailing vortices have two origins. First, there are vortex sheets at the trailing edges of the blades that are caused by the interaction, in the blade wakes, of the flows at different velocities over the suction and pressure surfaces of the blades. Also, there is the vortex sheet induced by a variable circulation over the blade height or by the decrease in circulation caused by a tip clearance at the hub or casing.

A vortex sheet does not persist in a real flow; it splits up into single vortices, which are unstable and tend to combine in larger single vortices [Ref. 10]. In the case presented in Fig. 1, single vortices are formed in the two halves of the blade passage with rotation contrary to the cascade secondary vortices. Due to the origins of the trailing vortices, their strength can be expected to be concentrated in the corner regions formed by the blade suction side and the end walls [Ref. 15]; hence, in actual flow measurements downstream of a blade row, it is unlikely that these vortices can be distinguished from the corner vortices.

D. TIP-CLEARANCE FLOW

Tip-clearance flow in unshrouded blading has two sources. The clearance space between the blade tips and the hub or casing allows a portion of the through flow to pass, which is undeflected and therefore does not participate in the energy exchange. Also, a tip vortex is formed due to the flow over the tip caused by the pressure difference on the suction and pressure sides of the blades.

E. SCRAPING EFFECT

The NACA smoke studies [Refs. 3, 6, and 7] demonstrated that when relative motion exists between the turbomachine blades and the casing, the leading surfaces of the blades scrape up entrained fluid near the wall and impart a vortex motion to the fluid in this region. However, R. C. Dean [Ref. 17] suggested that it is the fluid moving with the rotor, not the blade itself, which scrapes up the casing boundary layer. In either case it is the motion of the rotor blade relative to the casing boundary layer, rather than the pressure difference across the tip, that is responsible for the scraping vortex indicated in Fig. 1.

In a compressor where the pressure surface of the rotor blade leads, the scraping effect improves flow conditions in the tip region by preventing flow on the pressure side from deflecting over the tip, and reduces the tendency toward flow separation in the trailing-edge region of the blade suction surface by aspirating low-energy fluid off the suction side [Refs. 3 and 6]. In a turbine, on the other hand, where the rotor-blade suction surface leads, the scraping effect aggravates tip-clearance effects by decreasing the static pressure in the pressure-side tip region and by piling up fluid on and near the suction side, increasing the tendency for the flow to separate [Refs. 3 and 6].

F. RADIAL FLOW

Due to radial pressure gradients in annular cascades, boundary-layer material is transported along the blade span, mainly over the suction side in the rear part of the blade. For the stator the low-energy fluid is transported to the hub and produces larger losses in this region [Refs. 3 and 6]. In the case of the rotor, the centrifugal force can cause a radial flow from the hub to the tip of the blade [Ref. 13]. The amount of radial flow depends on the type of blade design.

Since only qualitative investigations of the scraping effect and radial flow have been conducted, the remainder of this report is only concerned with cascade secondary flow, corner vortices, trailing vortices, and tip-clearance flow. The above-described flow at the cascade exit presents two complicated problems to the turbomachine designer, the predictions of the radial or spanwise distribution of total-pressure loss and that of the average outlet flow angle.

III. SECONDARY-FLOW MODELS

Progress in the analytical treatment of secondary flows in axial-flow turbomachines has resulted from the development of four theoretical flow models: the inviscid model, the viscous model, the intermediate model, and the tip-clearance model.

A. THE INVISCID MODEL

The inviscid model for secondary flow in cascades, which is described in the section on cascade secondary flow, has received considerable attention in the past two decades [Refs. 17-48]. The designation inviscid is somewhat misleading; but it is used here to indicate that the flow through the passage, formed by two adjoining blades of the cascade and the end walls, is considered to be inviscid. The nonuniform velocity profile at the passage inlet is produced, of course, by viscous effects at the end walls. Due to the inviscid assumption, the cascade secondary flow can be treated theoretically.

There are two classical papers that pertain to the inviscid model, the one by Squire and Winter [Ref. 18] and the other by Hawthorne [Ref. 19]. The steady, incompressible, and inviscid flow (Fig. 2) is governed by the equation of motion

$$\nabla \times (\vec{V} \times \vec{\Omega}) = 0 \tag{4}$$

the continuity equation

$$\nabla \cdot \vec{V} = 0 \tag{5}$$

and the conservation of vorticity

$$\nabla \cdot \vec{\Omega} = 0 \tag{6}$$

where $\vec{V}$ is the velocity vector and $\vec{\Omega} = \nabla \times \vec{V}$ is the vorticity vector.

Squire and Winter [Ref. 18] considered the flow past a cascade for which the degree of nonuniformity of the inlet velocity distribution was small, so that the two-dimensional basic flow could be regarded as a first approximation to the actual flow. The actual flow was then calculated by treating it as a linear perturbation of the two-dimensional flow. This limits the calculation method to a flow passage with constant cross section and to small turning angles. An additional assumption of small passage width compared to bend radius further limits the analysis to bends with large aspect ratios h/s, for h and s as shown in Fig. 2. For an assumed free-vortex basic flow, the secondary vorticity at the cascade exit was found to be

$$\xi_2 = 2\Delta\alpha dV_1/dz \tag{7}$$

where ξ_2 is the outlet component in the flow direction of the vorticity vector $\vec{\Omega}$, $\Delta\alpha$ is the total deflection angle, dV_1/dz is the inlet velocity gradient, and z is the coordinate along h. The subscripts 1 and 2 refer to inlet and outlet conditions respectively. The quantity ξ_2 is the magnitude of the vectors $\vec{S}_u$ and $\vec{S}_l$ of Fig. 2. Squire and Winter also calculated the secondary-flow streamlines at the exit of a passage formed by two constant-thickness turning vanes, and obtained good agreement with their experimental data.

Hawthorne [Ref. 19] later developed a general theory for secondary flow, utilizing the inviscid model and Eqs. (4) through (6), to show that

$$(\xi_2/V_2) - (\xi_1/V_1) = -2\int_1^2 |\nabla P_t| \frac{\sin\theta}{\rho V^2} \, d\alpha \tag{8}$$

where ∇P_t is the total pressure gradient and θ is the angle between the directions of the principal normal to a streamline, which is in the direction of curvature, and of the normal to a constant-total-pressure surface, a Bernoulli surface, on which the streamline lies. The above expression is for the change in the streamwise component of vorticity or cascade secondary vorticity, and the integral is taken along a streamline. Equation (8) reduces to Eq. (7) for the simplifying assumptions made by Squire and Winter.

Loos and Zwaaneveld [Ref. 23], among others [Ref. 13], integrated Eq. (8) by assuming that $\xi_1 = 0$ and $\theta = \pi/2$ to get

$$\xi_2 = - \frac{dV_1/dz}{\cos \alpha_1 \cos \alpha_2}[(\alpha_2 - \alpha_1) + \tfrac{1}{2}(\sin 2\alpha_2 - \sin 2\alpha_1)] \tag{9}$$

where α_1 and α_2 are the fluid inlet and outlet angles respectively. Equation (9) is for constant axial velocity through the cascade. Hawthorne [Ref. 34] and Loos [Ref. 35] included the effect of compressibility on the cascade secondary flow, and for a perfect gas obtained

$$[\xi_2/(\rho_2 V_2)] - [\xi_1/(\rho_1 V_1)] = -2\int_1^2 |\nabla P_t| \frac{\sin \theta}{\rho_t \rho V^2}\, d\alpha \tag{10}$$

where ρ_t is the stagnation density.

Loos [Ref. 26] modified Eq. (7) for diverging or converging flow through a cascade, and the resulting expression is given by

$$\xi_2 = \frac{-[k - (1/k)]}{\ln k} \Delta\alpha dV_1/dz \tag{11}$$

where k is the ratio of the exit area to the entry area (normal to the flow). Since the limit of [k - (1/k)]/ln k is two for k approaching unity, Eq. (11) reduces to Eq. (7) for a constant-cross-section channel. Based on the work of Squire and Winter, Ehrich [Ref. 30] carried out a general analysis of the secondary flow in a cascade of blades whose properties vary as a function of blade height, i.e., twisted blades.

A. G. Smith [Ref. 38] extended the inviscid model to the secondary flow in rotating passages by expressing Eqs. (4), (5), and (6) in terms of a coordinate system that rotates with a constant angular velocity ω about a fixed axis, and he obtained

$$(\xi_2/W_2) - (\xi_1/W_1) = 2\int_1^2 |\nabla P_{tR}| \frac{\cos \gamma}{\rho W^2 R} d\beta + 2\int_1^2 |\nabla P_{tR}| \frac{\omega \cos \sigma}{\rho W^3} d\beta \tag{12}$$

which is an expression for the change in the streamwise component ξ of the absolute vorticity vector $\vec{\Omega}$. In Eq. (12) W is the relative velocity, P_{tR} is the relative total pressure $(P + \rho W^2/2 - \rho U^2/2)$, R is the principal radius of curvature of a relative streamline, γ is the angle between the principal normal to a relative streamline and the vector $(\nabla P_{tR}/\rho) \times \vec{W}$, σ is the angle between the vectors $(\nabla P_{tR}/\rho)$ and $\vec{\omega}$, and β is the distance along a relative streamline. Equation (12) can only be integrated for simplified cases and was not compared with experimental data in Ref. 38.

The cascade secondary vorticity described by Eqs. (7) through (11) is developed in the passage between the blades, but it is not the only vorticity that exists in the flow downstream of the cascade. Preston [Ref. 28] used the inviscid model and his concept that the vortex filaments carried by the rotational flow can be followed in the Lagrangian fashion, as the flow passes through a cascade, to describe three distinct elements of secondary vorticity: the cascade secondary vorticity, the trailing filament vortices, and the trailing shed vortices. The trailing filament vortices are caused by the stretching of the vortex filament, which is initially perpendicular to the entry stream, in the blade wakes. The stretching is due to the different velocities over the suction and pressure surfaces of the blades. The trailing shed vortices are due to the variation in circulation along the blade height.

In an analytical proof of the existence of these different elements of vorticity, Hawthorne [Ref. 31] derived the circulations associated with the cascade secondary vorticity, which is given by

$$\Gamma_1 = \frac{dV_1}{dz}\left(-V_1\oint \frac{dx}{V} + \frac{s}{2}\,\frac{\sin 2\alpha_2 - \sin 2\alpha_1}{\cos \alpha_1}\right) \qquad (13)$$

the trailing filament vortices, given by

$$\Gamma_2 = \frac{dV_1}{dz} V_1 \oint \frac{dx}{V} \qquad (14)$$

and the trailing shed vortices, given by

$$\Gamma_3 = -s \cos \alpha_1 \frac{dV_1}{dz}(\tan \alpha_2 - \tan \alpha_1) \qquad (15)$$

where x is the chordwise distance along the blade and s is the blade spacing. The arithmetical summation of these circulations has some physical significance since the change in the mean outlet flow angle can be calculated from the total circulation [Ref. 13].

In the above secondary-flow solutions, a small-shear, large-disturbance approximation has allowed a theoretical treatment of the flow. Honda [Ref. 42] utilized a large-shear, small-disturbance approximation to treat the secondary flow in cascades with the assumptions of thin-airfoil theory. Since the equations of motion can only be linearized if the disturbance is assumed to be small, the Honda theory is limited to cascades of thin blades with small deflections. Despite this limitation Honda obtained fair agreement between calculated values of the spanwise distribution of local lift coefficient using this theory and the values measured by Hawthorne and Armstrong [Ref. 32] in an impulse turbine cascade.

The simplifying assumptions applied in the development of the inviscid model restrict its use to the description of the secondary flow in cascades of turbomachine blades with high aspect ratios, the ratio of blade height to chord h/c, and small camber angles. However, it is clear from the analyses of the inviscid model that the generation of the component of vorticity in the direction of flow produces the cascade secondary flow.

B. THE VISCOUS MODEL

The inviscid model cannot be used to describe the secondary flow near the end walls that bound the cascade blades, because of the influence of the boundary layers that are present in these regions. The viscous model, as depicted in Fig. 3, allows for the effects of fluid viscosity inside the blade passage and for the formation of the corner vortices. The importance of the viscous model is confirmed by the experiments of Refs. 3, 6, and 12, which showed that the flow in the corner between the suction surface of the blade and the end wall has a tendency to separate due to the interaction of the corner vortices with the main flow. Consequently, viscous effects are dominant in this region of high secondary loss.

A complete theoretical solution of the viscous model is not possible at the present time. The problem of calculating the three-dimensional, asymmetric, compressible, and turbulent boundary-layer flow in turbomachines is one of the most difficult in engineering fluid mechanics [Ref. 49]. The efforts that have been made to analytically treat the viscous model are discussed in the following paragraphs.

Loos [Ref. 26] developed a modified linearized theory which takes into account the corner vortices in an approximate manner. Hawthorne [Ref. 43] and Marris [Ref. 50] included viscous effects by extending the analysis of the inviscid model. Louis [Ref. 51] used the superposition of two two-dimensional flows, a straight viscous flow and a potential flow with slightly curved streamlines, to develop a viscous fluid theory for secondary flow.

The references of the preceding paragraph, however, do not attempt to calculate the boundary-layer development on the end wall of the blade passage. The three-dimensional boundary-layer flow in internal aerodynamic configurations, e.g., turbomachines, differs from that in external aerodynamic configurations, e.g., airplane wings, in the nature of the flow outside the boundary layer. In the former the flow outside the boundary layer may be rotational, while in the latter that flow is potential or irrotational. For the viscous model of secondary flow, the total flow field can then be divided into a region where the effects of viscosity are neglected and a thinner boundary region where such effects are considered.

In Ref. 17 R. C. Dean presented a laminar boundary-layer analysis of secondary flow using a momentum-integral technique, but he did not achieve satisfactory agreement between his theoretical and experimental results. Mager [Ref. 52] resolved the flow outside the boundary layer into a large principle component and a small crosswise velocity to develop a small-perturbation analysis of the three-dimensional laminar boundary layer, which is restricted to regions of low mainstream turning. In order to get boundary-layer solutions that are not limited to small main-streamline deflection, Hansen and Herzig [Refs. 8 and 9] extended the analysis of Ref. 52 by assuming laminar main-streamline translates, i.e., the entire streamline pattern can be obtained by translating any particular streamline in a direction parallel to the leading edge of the surface, corresponding to the assumption of axial symmetry in turbomachine design.

Senoo [Ref. 53] developed a theory for the three-dimensional laminar boundary layers on the plane end walls of curved, accelerating channels. In an investigation of the boundary layer on the end wall of a turbine stator cascade, Senoo [Ref. 16] was able to show that the boundary layer was laminar at the stator throat; and he obtained good agreement between his theory and experimental data in the flow region where the blade surfaces could be approximated by a pair of logarithmic spiral lines.

The turbulent end-wall boundary layer has been treated by Lewkowicz for the flow in a bend, where the flow angle distribution through the boundary layer was successfully predicted. This theory, which uses empirical streamwise and cross-flow velocity profiles in the streamwise and normal momentum-integral equations, is discussed in Ref. 49. In Ref. 54 Mellor and Wood presented an end-wall boundary-layer theory for axial-flow compressors based on an equilibrium-stage hypothesis, i.e., the casing boundary layer in a multistage compressor will grow to some asymptotic size and remain constant thereafter. L. H. Smith [Ref. 55] obtained experimental evidence that indicates that this repeating-stage flow model is a reasonably accurate one; however, the theory of Ref. 54 has not been compared to experiment.

Gregory-Smith [Ref. 56] attempted to include the effects of the annulus-wall boundary layers in a calculation of the flow in an axial turbomachine, but he found that the boundary-layer development was not predicted with sufficient accuracy in his theory. The work of Gregory-Smith has shown that an improved theory based on the viscous model of secondary flow can result in reliable calculations of the complete flow patterns in axial turbomachines.

As previously stated, a complete analytical threatment of the viscous model is not presently available due to the complex nature of this secondary-flow model. The theories presented above are not able to account for the interaction of the blade suction-surface boundary layer and the end-wall boundary layer, for the formation of the corner vortices, nor for the effects of three-dimensional boundary-layer separation. Railly and Howard [Ref. 57] tried to include the separation effects on the secondary flow in compressors in an approximate fashion using a so-called stall-zone theory, but their calculations did not compare well with experiment.

C. THE INTERMEDIATE MODEL

In Ref. 58 Traupel proposed a theoretical model of secondary flow, which is designated as the intermediate model since the boundary-layer development on the end walls inside the blade passage is described by this model but not the interaction of the suction-surface and end-wall boundary layers. Traupel represented the flow in cascades as a vortex model consisting of a primary vortex system and a secondary vortex system.

The boundary layers that are produced at the walls enclosing the flow of a viscous fluid can be considered as a system of vortex lines or filaments superposed on a flow without boundary layers, as shown in Fig. 4 for the flow between two parallel walls of a channel. The deflection of such a parallel flow in a cascade of blades results in the primary vortex system. This system of free vortices has the same significance as the concept of bound vorticity that is introduced into airfoil theory to represent circulation. The deflection of the vortex lines that are normal to the flow produces a component of vorticity in the direction of the main flow and forms the secondary vortex system. The resulting secondary flow at the cascade exit, consisting of the cascade secondary vortices S_u and S_l and the trailing vortices T_u and T_l, is schematically represented in Fig. 5.

Only qualitative information on secondary flow can be obtained with Traupel's model [Ref. 10]. However, Griepentrog [Ref. 2] used the horseshoe vortex replacement for a wing of finite span, which is composed of a principal vortex and two trailing vortices, as an approximation of the intermediate model. The principal vortex produces lift, and the velocities induced by the trailing vortices cause an additional drag. In Ref. 2 the cascade of blades is replaced by a system of isolated airfoils, and the effect of the end walls is represented by a system of image airfoils. This model for the calculation of the increased drag due to secondary flow was first proposed by Carter and Cohen [Ref. 59].

The secondary drag coefficient as calculated by Griepentrog is

$$C_{D_s} = \frac{1}{4} C_L^2 \frac{c}{s} \frac{1 - h'/h}{\sqrt{1 - (1 - h'/h)^2}} \tag{16}$$

where C_L is the lift coefficient at the blade mid-height, c is the blade chord, s is the cascade spacing, h' is the height of the replacement blade, and h is the height of the original blade. The height of the replacement blade is also the distance between the trailing vortices, which is a function of the end-wall boundary-layer thickness and is given by

$$\frac{h'}{h} = 1 - \frac{4}{3} \frac{\delta}{h} \tag{17}$$

where δ is the boundary-layer thickness at the exit of the blade passage. The secondary drag coefficient C_{D_s} is related to the total pressure loss due to secondary flow ω_s by

$$\omega_s = C_{D_s} \frac{c/s}{\cos \alpha_m} \tag{18}$$

where α_m is the mean flow angle, $(\alpha_1 + \alpha_2)/2$. Griepentrog applied the method described above to compressor stator bladings with high deflection angles and small aspect ratios, and obtained a satisfactory correspondence between calculated and measured values of secondary loss coefficients.

D. THE TIP-CLEARANCE MODEL

The preceding secondary-flow models do not include the effects of tip-clearance flows. There are essentially two approaches that are used for the theoretical treatment of these effects. Betz [Ref. 60] replaced the cascade blades by lifting lines, and considered the velocities induced at the lifting line by the vortices shed along the blade span. Even though viscous effects inside the clearance space and the presence of an end wall are neglected, Meldahl [Ref. 61] applied this method with some success to the flow through turbine cascades. As a second approach to the problem, Rains [Ref. 62] and Vavra [Ref. 14] assumed that the leakage flow, resulting from the pressure difference across the tip, occurs in an annulus height equal to the clearance gap, and considered the kinetic energy associated with such a flow as tip-clearance loss. This method, however, neglects the interaction of the clearance flow and the tip vortex with the flow in the blade passage.

Lakshminarayana and Horlock [Ref. 63] formulated a modified lifting-line approach which is called the potential vortex model. In this model the lift is assumed to be uniform along the blade span, and only part of the bound vortex is assumed to be shed at the trailing edge of the blade tip. The boundary condition at the end wall is satisfied by the use of image vortices. An inviscid analysis is used to determine the induced velocities along the lifting line due to the tip and image vortices, and the tip-clearance loss is expressed as an induced drag coefficient.

However, this loss does not take into account the losses associated with the spanwise flow that occurs inside the blade boundary layer near the tip due to the large spanwise pressure gradients in this region. Based on the potential vortex model with an additional loss term for spanwise flow, Lakshminarayana [Ref. 64] derived a semi-empirical expression for the decrease in stage efficiency due to tip clearance, which is given by

$$\Delta\eta = \frac{0.7\lambda\psi}{\cos\alpha_m}\left[1 + 10\sqrt{\frac{\varphi}{\psi}\,\frac{\lambda A}{\cos\alpha_m}}\right] \qquad (19)$$

where $\Delta\eta$ is the decrease in stage efficiency based on total pressure rise or drop, λ is the nondimensionalized clearance height, ψ is the blade loading coefficient, α_m is the mean flow angle, φ is the flow coefficient, and A is the blade aspect ratio. In Ref. 64 the predictions obtained from Eq. 19 are compared with available experimental data for several axial-flow compressor, fan, pump, and turbine stages; and the agreement is quite good.

The potential vortex model provides sufficiently accurate results for the overall effect of tip-clearance flow in terms of efficiency loss, but this model is too simplified to predict the variation of flow properties from blade to blade. In Ref. 64 Lakshminarayana proposed a modified tip-clearance flow model, which is shown in Fig. 6, for the accurate prediction of the blade-to-blade flow in the tip region. The leakage flow that originates from the blade tip along the chord forms vortex layers that roll up into a spiral to form a core of rotating fluid, which lies near the suction surface of the blade and inboard of the blade tip. The vortex core is assumed to contain all of the shed vorticity in the field; hence, the fluid motion outside of the core is irrotational. Image vortices located at equal distances from the end wall are used to satisfy the boundary condition of zero normal velocity on the wall adjoining the clearance region. Using the modified tip-clearance model, Lakshminarayana obtained accurate predictions of the outlet flow angle but only qualitative indications of the nature of the associated secondary loss.

IV. SECONDARY-FLOW LOSSES

There are two methods for the calculation of the flow in turbomachines that are used for performance prediction, the streamline-curvature method and the matrix through-flow method. These methods, as described by Marsh [Ref. 65], are based on the same mathematical model but differ in their numerical solution techniques. One difficulty in calculating the overall flow pattern is that it is necessary to estimate the radial distribution of the loss in total pressure. In general the losses in total pressure through a blade row in a turbomachine are attributed to profile boundary layer, wake mixing, secondary flows, and shocks if present. The radial loss distribution is characterized by an increase in loss in the vicinity of the end walls due to secondary flow, as shown in Fig. 7 for a plane compressor cascade. As yet no theoretical method is available to predict the losses in total pressure caused by secondary flow.

The secondary-flow models treated in the preceding section give a physical description of secondary flow but no quantitative information on secondary losses. The inviscid model is restricted in its application and is limited to the calculation of the kinetic energy associated with the cascade secondary vortices, which only gives qualitative data on the losses due to secondary flow [Ref. 67]. L. H. Smith [Ref. 33] concluded that the secondary losses due to the trailing vortices produced by variable circulation are negligible for most practical cases. However, Lakshminarayana and Horlock [Ref. 13] indicated that these losses are of sufficient magnitude to be taken into account when large tip clearances are used, as in axial-flow fan stages. The viscous model qualitatively describes the formation of the corner vortices which cause large secondary losses, especially if the corner boundary-layer flow is separated. Equations (18) and (19), which are obtained from analyses of the intermediate model and the tip-clearance model respectively, must be regarded as semi-empirical expressions for the prediction of secondary losses.

Empirical or at best semi-empirical relations are used to account for the three-dimensional or secondary effects in the design of turbomachines. A comprehensive summary of these relations is contained in Ref. 13. Most secondary-flow analyses, experimental and theoretical, neglect the skewness of the inlet boundary layer [Ref. 68] and the inlet component of streamwise vorticity [Ref. 69], which exist in the actual flow through turbomachines and are likely to have a substantial effect on the secondary losses [Ref. 67].

V. OUTLET FLOW ANGLE

The spanwise variation in outlet flow angle caused by secondary flow can be qualitatively explained with the aid of Fig. 8, which is based on the intermediate model of secondary flow. At the blade end with no clearance, the cascade secondary vortex S_I is predominant, causing an increased deflection or overturning of the flow. At some distance from the end wall, a reduced deflection or underturning is experienced due to the effect of the trailing vortex T_I. As the distance from the wall increases and the center of the vortex T_I is passed, the deflection increases again to approach the two-dimensional value at the center of the blade. At the clearance end of the blade, the vortex T_{II}, which is augmented by the tip vortex, has a stronger influence than the cascade secondary vortex S_{II} causing an underturning of the flow.

The inviscid model of secondary flow has been more useful for the prediction of outlet flow angle than for the calculation of secondary losses. Carter [Ref. 70] and Hausmann [Ref. 71] considered only the effects of the trailing vortices. Since these analyses neglected the cascade secondary vortices, satisfactory agreement with experimental results was not obtained.

Hawthorne and Armstrong [Ref. 72] and Gomi [Ref. 73] included the secondary motions that are induced by the distributed vortices in the flow between the blades. In Ref. 72 the inviscid model and actuator-disc theory were used to predict the change in average outlet flow angle α_2, which is given by

$$\Delta\alpha_2 = - \frac{2 \cos \bar{\alpha}_{02}}{V_1 \cos \alpha_1} \sum_n \frac{\psi_n'}{n} \tag{20}$$

where $\bar{\alpha}_{02}$ is the angle at which the flow leaves the cascade trailing edge inside the uniform region, V_1 and α_1 are the inlet velocity and flow angle respectively, and ψ_n' is the spanwise partial derivative of the secondary stream function. Gregory-Smith [Ref. 56] used an analysis similar to the above to include the radial variation of outlet flow angle in his calculation method for the complete flow patterns in turbomachines.

Lakshminarayana and Horlock [Ref. 74] extended the analysis of Ref. 72 to include the effects of Bernoulli-surface rotation in the blade passage and of viscous decay of the inlet velocity profile up to the actuator plane. These modifications increased the accuracy of this prediction method as evidenced by the good agreement obtained between experiment and theory in Ref. 74. For cascades with tip clearance, Lakshminarayana [Ref. 64] used his modified clearance model to obtain the average change in outlet flow angle at any spanwise position, which can be approximated by

$$\begin{aligned} \Delta\alpha_2 &= \arctan\left[0.25\tfrac{c}{s}\, C_L\left(1 - \frac{y - \tau}{a}\right)\right] \text{ for } a + \tau \geq y \geq \tau \\ \Delta\alpha_2 &= 0 \quad \text{for } y > a + \tau \end{aligned} \qquad (21)$$

where c and s are the blade chord and spacing respectively, C_L is the lift coefficient based on the mean velocity, y is the spanwise coordinate, τ is the clearance height, and a is the radius of the vortex core. In comparison with available experimental data [Ref. 64], Eq. (21) gave reasonably accurate results.

The above predictions for the variation in outlet flow angle due to secondary flow are not valid for high-deflection cascades, greater than 80 degrees, nor in the presence of corner separation. Also, the effects of the skewness of the inlet boundary layer and of a streamwise component of vorticity at the inlet are neglected.

VI. CONCLUSIONS

Because of the complex nature of the flow in axial-flow turbomachines, no theoretical method is available to predict the losses in total pressure that are associated with secondary flows. The type of flow can be predicted but not the losses. Inviscid analyses have been successfully used to calculate the distribution of the average outlet flow angle along the blade height.

More general correlations, empirical or semi-empirical, that are compatible with fundamental physical principles must be developed for application to the improved design of the modern turbomachine. Important real-flow effects, such as the skewness of the inlet boundary layer and the inlet component of streamwise vorticity, should be considered in the analysis of secondary-flow losses.

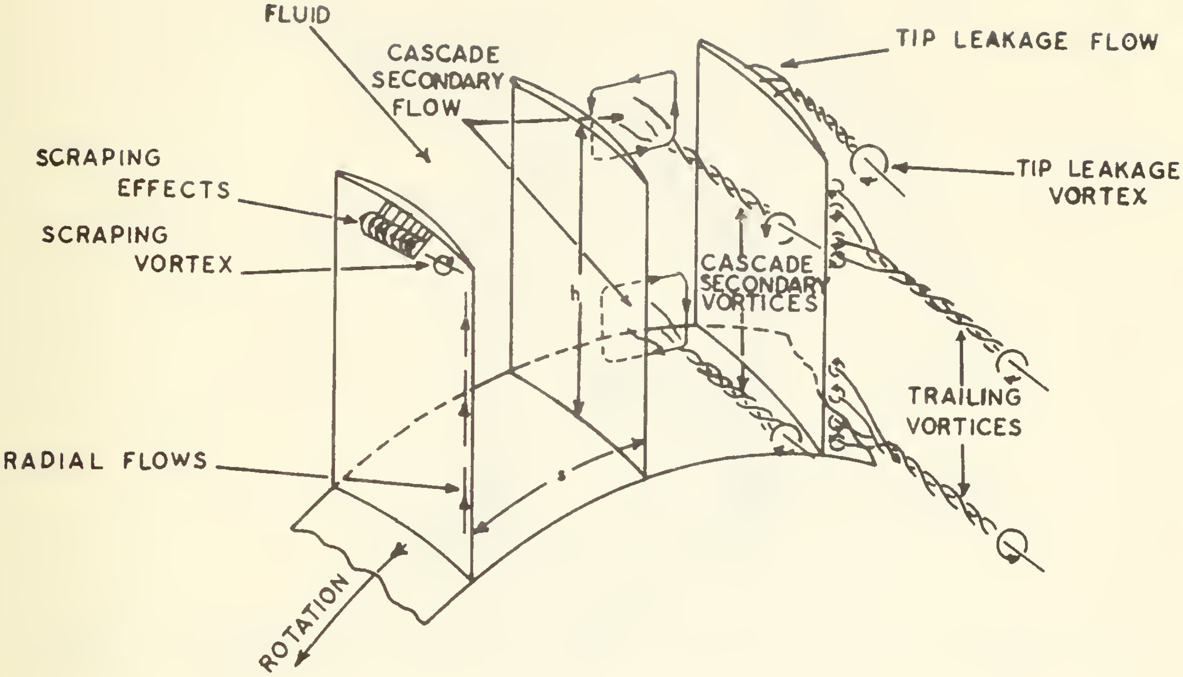

FIG. I SECONDARY FLOWS AND VORTICES IN AN AXIAL-FLOW COMPRESSOR ROTOR (adapted from Ref. 13)

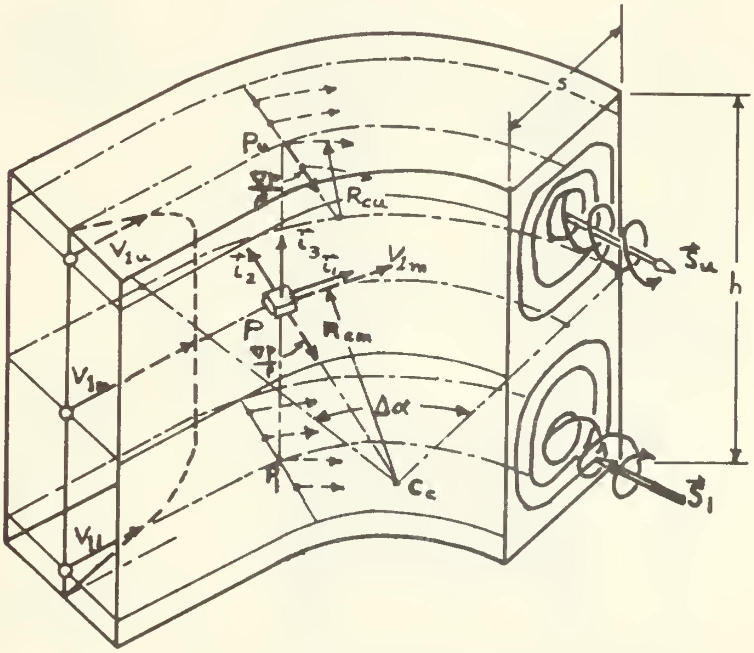

FIG. 2 SECONDARY FLOW IN A BEND (adapted from Ref. 14)

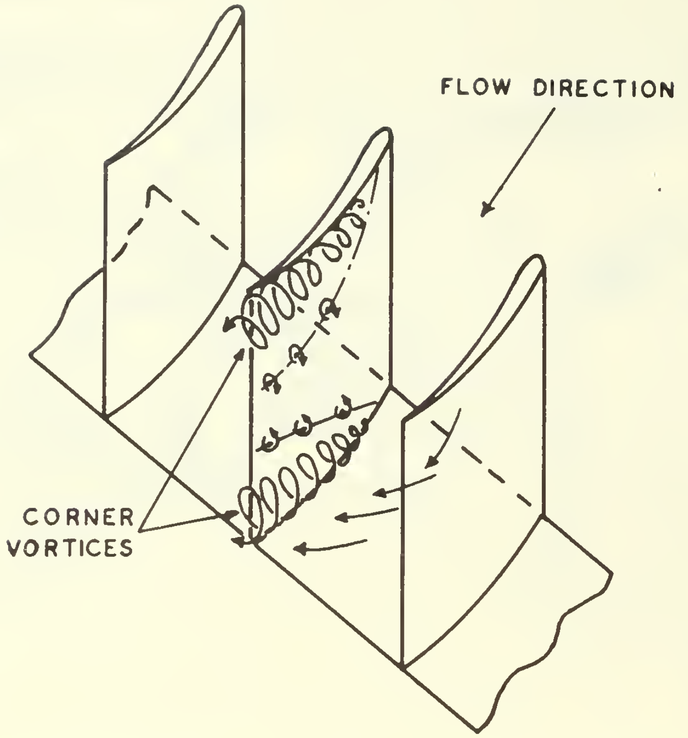

FIG. 3 THE VISCOUS MODEL SHOWING THE CORNER VORTICES (adapted from Ref. 15)

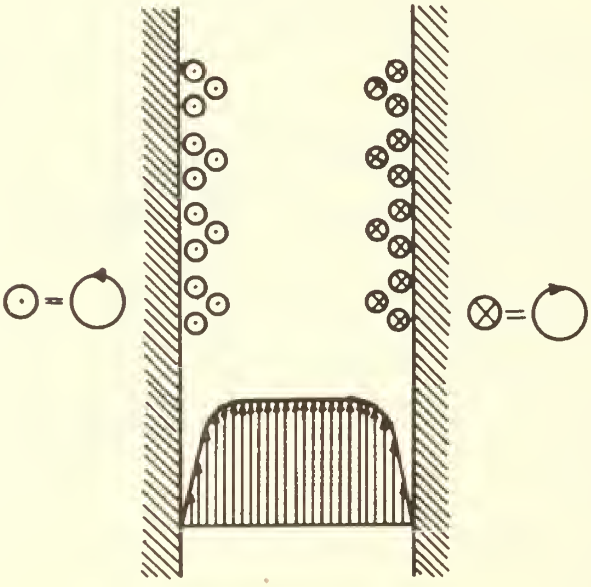

FIG. 4 VORTEX SYSTEM FOR THE VISCOUS FLOW BETWEEN TWO PLANE PARALLEL WALLS (reproduced from Ref. 58)

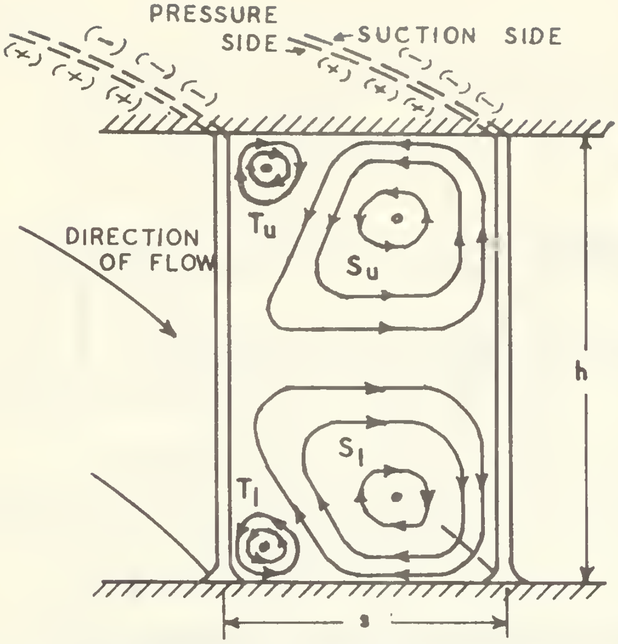

FIG. 5 VORTEX PATTERN AT EXIT OF CASCADE (adapted from Ref. 14)

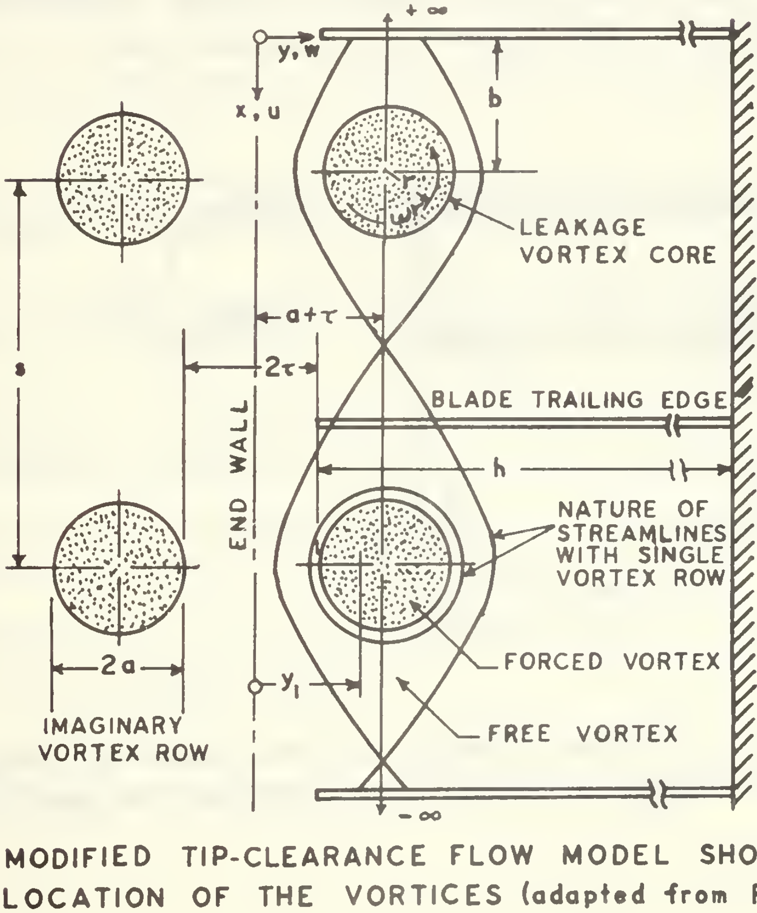

FIG. 6 MODIFIED TIP-CLEARANCE FLOW MODEL SHOWING THE LOCATION OF THE VORTICES (adapted from Ref. 64)

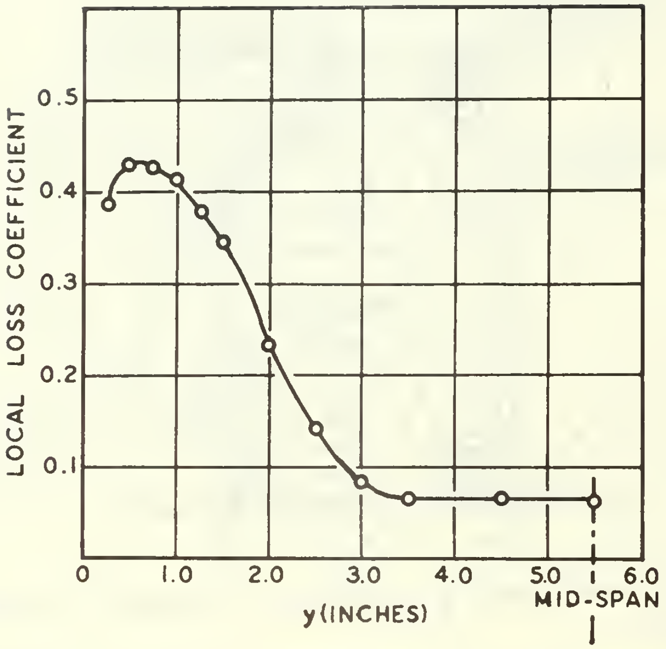

FIG. 7 TYPICAL LOSS DATA FROM A PLANE COMPRESSOR-CASCADE TEST (reproduced from Ref. 66)

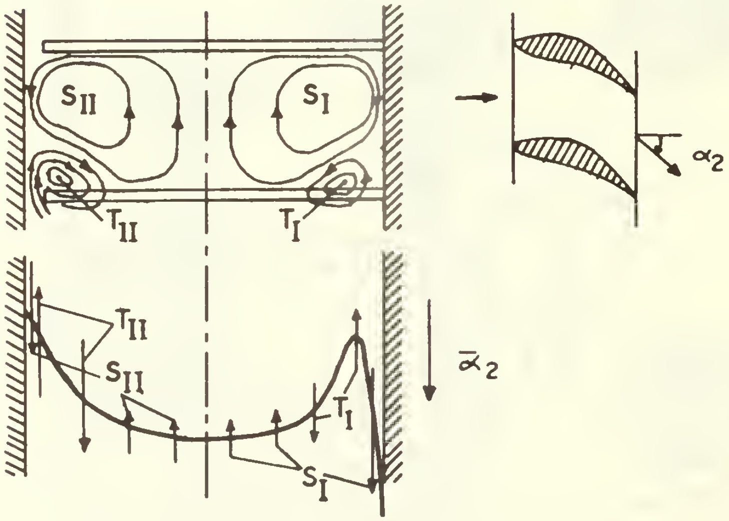

FIG. 8 EFFECT OF THE VORTEX SYSTEMS ON THE DEFLECTION OF THE FLOW IN A CASCADE (adapted from Ref. 10)

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13 ABSTRACT

This report presents the current state of the art of the analytical treatment of secondary flows and associated losses in axial-flow turbomachines. A physical description of the secondary-flow phenomena is given, as well as the theoretical methods that are available to treat secondary flows and to predict secondary losses.

14 KEY WORDS	LINK A		LINK B		LINK C	
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Axial-Flow Turbomachines						
Secondary-Flow Models						
Secondary-Flow Losses						
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